# Interpretation of VLBI Results in Geodesy, Astrometry and Geophysics

# Combination of Nutation Time Series Derived from VLBI and GNSS

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**Abstract.** In this work we concentrate on the combination of 2-years (2005-2006) series of nutation offsets from VLBI and nutation rates from GNSS obtained w.r.t. the IAU 2000 nutation model. In order to take advantage of both series the combined smoothing method introduced by Vondrák and Čepek has been used. From combined series amplitude corrections to the IAU 2000 model at selected frequencies have been estimated.

#### 1. Introduction

VLBI is the unique space geodetic technique which provide a direct access to the high-stable celestial reference frame, based on extragalactic objects (ICRF). Therefore, only VLBI allows to estimate nutation offsets with a long-term stabilit straightforwardly. In case of GNSS (Global Navigation Satellite System) technics measurements of the Earth orientation are carried out w.r.t. the satellite orbits. Due to the high correlations between some EOP (nutation angles and UT1-UTC values) and orbital parameters [1], the satellite methods can provide only the first time derivatives of those EOP (i.e. nutation rates  $\Delta \dot{\psi} \sin \varepsilon_0$ ,  $\Delta \dot{\varepsilon}$  and LOD). And even in this case, nutation rates can be estimated only over the time interval where satellite orbits can be modeled accurately enough, i.e. 1-3 days. Nevertheless, it is expected that GNSS observations can contribute essentially to the determination of nutation parameters in the high frequency domain (periods from 5 to 28 days).

# 2. Strategy of Nutation Offsets/Rates Estimation from VLBI and GPS Observations

The VLBI solution has been obtained by the OCCAM v.6.1 software using the least squares method (LSM). In order to produce this solution the celestial as well as the terrestrial reference frames have been fixed to ICRF-Ext.2 catalog and ITRF 2005, respectively. Nutation offsets have been estimated w.r.t. the IAU 2000A nutation model. Diurnal and semidiurnal EOP variations according to the IERS Conventions 2003 [2] have been taken into account.

Nutation offsets  $(\Delta \psi, \Delta \varepsilon)$  as well as other EOP have been modeled as linear functions. The parameters offset and rate (both fully unconstrained) have been estimated once per session. Troposphere parameters and station clocks have also been estimated in this solution. We used all 24-hours VLBI sessions within the interval from Jan. 1, 2005 to Dec. 31, 2006 for obtaining of this solution.

In order to obtain nutation rates from GPS observations we used the Bernese v.5.0 software [3]. As a priori information IGS precise orbits and the radiation pressure model developed by the Center for Orbit Determination in Europe (CODE) were taken. All calculations have been made for a network of 116 globally distributed stations which are a part of IGS realization of the ITRF 2005. Nutation rates have been estimated w.r.t. the IAU 2000 nutation model. For diurnal and semidiurnal EOP variations IERS Conv. 2003 model have been applied.

To calculate nutation rates series we have to establish very stable satellite orbits. Therefore the orbits have been characterized by one set of Keplerian elements and corresponding radiation pressure parameters once per three days arc. We estimated corrections to the tracing station positions under NNR condition. Pole coordinates and LOD have been estimated once per 24-hours session and nutation rates — once per 3-days solution (related to the middle day of this 3-days interval). Initial phase ambiguities as well as site-specific troposphere parameters have been pre-eliminated before the LSM adjustment.

As a result we come up with a homogeneous (from the point of view of processing strategy) time series of nutation rates covering the period from Jan. 1, 2005 to Jul. 29, 2006.

# 3. Analysis of Uncertainties

The uncertainties of nutation rates clearly show some systematic effects (Fig. 1a). Spectral analysis of the uncertainties revealed three main periodical terms with periods of about 60, 90, and 180 days. It seems that the 180 days period stems from the correlations of satellite orbits at long time scales with pressure radiation parameters. The terms with periods of about 60 and 90 days could be explained by the precession of GPS orbits (see Rothacher et. al., this volume).

The uncertainties of VLBI solution do not show any systematic effects, but there are some sessions in 2006 with larger errors (Fig. 1b). The quality of nutation offsets could be affected by the geometry of the VLBI network which changes from one session to another according to the stations participating in a particular experiment.

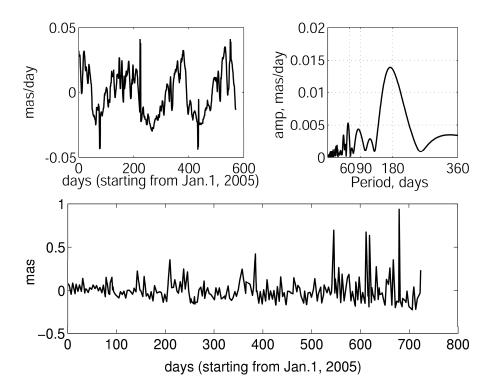


Figure 1. Upper, left: uncertainties of GPS based nutation rates in longitude; right: amplitude spectrum of uncertainties reveal peaks with the periods of about 60, 90, and 180 days. Lower: uncertainties of VLBI based nutation offsets in longitude

#### 4. Combination Method

In order to combine the two series described above we applied the Vondrák combined smoothing method [4]. The method has been developed for combining the function values F and their first time derivatives  $\tilde{F}$ . A solution is looked for in a form of smooth curve (with smoothness S) which fits both observed series. For this purpose the following weighted sum has to be minimized:

$$Q = S + \varepsilon F + \tilde{\varepsilon}\tilde{F} = min, \tag{1}$$

where  $\varepsilon, \tilde{\varepsilon} \geq 0$  are the smoothing coefficients expressing the relative importance of each item: the larger the value of the coefficients the closer is the smoothed curve to the observations. The choice of numerical values of these coefficients  $\varepsilon = \left(\frac{12.6}{P_{min}}\right)^6 \approx 34$ ,  $\tilde{\varepsilon} = \left(\frac{16.9}{P_{min}}\right)^4 \approx 34$ , is based on the conclusions given in [4, 5].

Such a choice of numerical values of smoothing coefficients assures that the

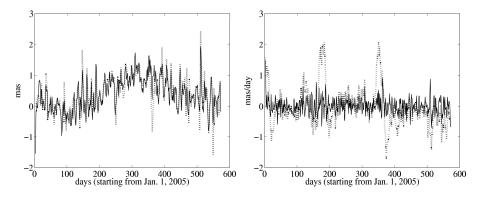


Figure 2. Combined (solid line) and original (dotted line) time series of nutation in longitude

smoothness is weak enough in order to keep the signals with periods longer than 7 days unchanged and at the same time is sufficiently strong in order to suppress all periods shorter than 2 days (i.e. high frequency noise of the observations). The combined series are presented in Fig. 2.

In Tabl. 1 the a posteriori uncertainties, computed from residuals (between observed and smoothed values), are compared with the a priori values. The table clearly shows that the a posteriori values of uncertainties for nutation rates  $(\Delta \dot{\psi} \sin \varepsilon_0, \Delta \dot{\varepsilon})$  are about five times larger than the a priori values. Rates uncertainties are underestimated due to the fact that we take into account only the random part of the errors but not the systematic part which is caused by difficulties in orbit modeling.

# 5. Estimation of Amplitude Corrections w.r.t. the IAU 2000 Nutation Model from Combined Series

Since the main impact from the combination of GPS and VLBI series in comparison with VLBI series has been expected only in high frequencies, we estimated corrections to 10 nutation terms with periods from 4.684 days to 27.555 days. From the combined nutation offset series amplitude corrections to the IAU 2000 nutation model have been estimated in according to:

Table 1. A priori and a posteriori uncertainties

uncertainties	$\Delta\psi\sin\varepsilon_0$ ,	$\Delta \dot{\psi} \sin \varepsilon_0$ ,	$\Delta \varepsilon$ ,	$\Delta\dot{arepsilon},$	
	$\max$	$\max$	$_{ m mas}$	$_{ m mas}$	
a priori	0.231	0.066	0.096	0.029	
a posteriori	0.140	0.455	0.054	0.149	

$$\Delta\psi\sin\varepsilon_0 = a + bt + \sum_{i=1}^{10} A_i^{in}\sin(arg) + A_i^{out}\cos(arg), \qquad (2)$$

$$\Delta \varepsilon = c + dt + \sum_{i=1}^{10} B_i^{in} \cos(arg) + B_i^{out} \sin(arg), \tag{3}$$

where a,b,c,d – parameters of linear trends,  $A_i^{in},B_i^{in}$  – in-phase components,  $A_i^{out},B_i^{out}$  – out-of-phase components,  $arg=\sum_{j=1}^5 N_j F_j$  with the Delaunay variables  $F_j$ . The results are presented in Tabl. 2.

Table 2. Estimated amplitude corrections w.r.t the IAU 2000 nutation model from the combined series

Periods,	comp.	Ampl.,	$\sigma$ ,	Periods,	comp.	Ampl.,	$\sigma$ ,
day		$\mu{ m as}$	$\mu as$	day		$\mu{ m as}$	$\mu as$
	$A^{\imath n}$	1	34		$A^{\imath n}$	-1	34
4.684	$A^{out}$	10	34	10.085	$A^{out}$	-12	34
	$B^{in}$	0	15		$B^{in}$	6	14
	$B^{out}$	27	14		$B^{out}$	-21	14
	$A^{in}$	-37	34		$A^{in}$	11	34
5.492	$A^{out}$	-3	34	13.661	$A^{out}$	-128	34
	$B^{in}$	-2	14		$B^{in}$	42	14
	$B^{out}$	-9	14		$B^{out}$	15	14
	$A^{in}$	-15	34		$A^{in}$	-20	34
5.802	$A^{out}$	22	34	14.765	$A^{out}$	132	34
	$B^{in}$	-10	14		$B^{in}$	-25	14
	$B^{out}$	-16	14		$B^{out}$	-33	14
	$A^{in}$	31	34		$A^{in}$	-35	34
7.096	$A^{out}$	57	34	23.942	$A^{out}$	-41	34
	$B^{in}$	-2	14		$B^{in}$	-3	14
	$B^{out}$	29	14		$B^{out}$	-15	14
	$A^{in}$	27	34		$A^{in}$	112	34
9.133	$A^{out}$	-62	34	27.555	$A^{out}$	139	34
	$B^{in}$	-7	14		$B^{in}$	-13	14
	$B^{out}$	-24	14		$B^{out}$	19	14

### 6. Conclusions

It has been shown that GPS based nutation rates suffer from large long periodic systematic effects with amplitudes of about 0.02 mas/day. Spectral analysis of the uncertainties reveals three main periods of about 60, 90 and 180 days. All these periods can be explained by correlations between the estimated parameters (radiation pressure parameters and orbital elements) and by a mis-

modeling of GPS satellite orbits at long time periods. Due to the fact that we do not take into account this systematic part of the errors, the values of nutation rate uncertainties seem to be highly underestimated.

Since we did not perform a calibration of GPS based nutation parameters to VLBI based values, systematic errors mentioned above could propagate into the combined series. Consequently, these effects could distort estimated amplitude corrections, even of high-frequency terms.

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